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Gluon matter plasma in the compact star core within fluid QCD model

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Abstract The structure of compact star core filled by gluon matter plasma is investigated within the fluid-like QCD framework. The energy momentum tensor, density and pressure relevant for gluonic plasma having the nature of fluid bulk of gluon sea are derived within the model. It is shown that the model provides a new equation of state for perfect fluid with only a single parameter of fluid distribution $\phi(x)$. The results are applied to construct the equation of state describing the gluonic plasma dominated compact star core. The equations of pressure and density distribution are solved analytically for small compact star core radius. The phase transition of plasma near the core surface is also discussed.

Keywords quark-gluon-plasma · fluid QCD · compact star · equation of state

1 Introduction

Recent experiments in the last decades on relativistic nuclear collisions shed light on the phenomena of hot plasma formed by dense quarks and gluons. Those experiments suggest that the quark gluon matter behaves more like a deconfined quark-gluon plasma (QGP) liquid [1–3].

This fact immediately encourages some models based on either the (relativistic) hydrodynamic [4,5], or pure Quantum Chromodynamics (QCD) approaches [6]. Within the hydrodynamic framework, the plasma is dominated by either quark [5] or gluon matter [4]. In particular, in the quark matter dominated plasma dissipative ideal hydrodynamics has been used to fit some experimental data at high energy heavy ion program at the Relativistic Heavy

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Ion Collider (RHIC) [7]. The successful fit requires the models to take into account very small value of the ratio shear viscosity over entropy [8–13]. However the puzzle must still be confirmed by the next coming experiments at the Large Hadron Collider (LHC) [14]. While the gluon matter plasma is motivated by the discoveries of jet-quenching in heavy-ion-collision at RHIC indicating the shock waves in form of March cones [15,16].

On the other hand, in pure QCD, QGP is described as a quark soup before hadronization which is a phase of QCD, and exists at extremely high temperature and/or density. It is argued that this phase consists of almost free quarks and gluons. Therefore, the phase transition from the deconfined QGP to the hadronic matters or vice versa gets particular interest in this approach. It unfortunately turns out to the many body problems with large color charge which cannot be calculated analytically using perturbation. As a result, the main theoretical tools to explore QGP within QCD is lattice gauge theory. The lattice calculation predicts that the phase transition occurs at approximately 175 MeV [17,18]. Many works on both analytical and numerical calculations have been done at a sophisticated level providing some interesting predictions like color glass superconductivity (CGS), color-flavor locked (CFL) phase and so on.

Anyway, describing the QGP as either quark or gluon matter dominated plasma within hydrodynamic or pure QCD should be understood as conjectures in the world of QGP. The important point is, since the QGP contains many quark-anti-quarks and gluons, it is considerable to treat it using the well-established QCD. At the same time from hydrodynamic point of view, the experiments strongly suggest that QGP behaves like a fluid. In particular, in the scenario of viscous gluonic plasma this is required to form and propagate shock waves [19–21]. Moreover it dissolves into an almost perfect dense fluid [22]. Therefore it is plausible to describe it as a strongly interacting fluid system. In this sense, there are approaches based on unifying or hybridizing the charge field with flow field [23–28]. Recently, some works have constructed the models in a lagrangian with certain non-Abelian gauge symmetry to the matter inside the fluid [29,30].

In this paper, we follow the scenario of gluon matter plasma as suggested by hydrodynamic approach, while it is still governed by QCD. In this framework, the strongly interacting system of QCD is considered as a macroscopic fluid system rather than a result of many particles interactions. It is shown that one can derive the total energy momentum tensor of gluon plasma within the model. Further, the equation of state (EoS) relevant for plasma dominated compact star core interiors is constructed. In the model, the density (ρ) and pressure (p) are determined and related each other through the fluid field distribution $\phi(x)$. Hence, the EoS can be solved through the ordinary differential equation (ODE) of ρ to show the phase transition within the model.

The paper is organized as follows. First we briefly introduce the underlying model of gauge invariant fluid lagrangian and discuss the relevant physical scale and region within the model. Then, the energy momentum tensor, density and pressure in the model are derived and investigated. Subsequently it

is followed by constructing the relevant EoS for compact star core interiors. Before summarizing the paper, the analytical solution for the pressure and density distributions of small compact star core radius is given and discussed.

2 The model

Let us adopt the model developed by Sulaiman *et.al.* [30,31]. The model proposes to describe the QGP as a strongly interacting gluon sea with quarks and anti-quarks inside. The model deploys the conventional QCD lagrangian with SU(3) color gauge symmetry, that is,

$$\mathcal{L} = i\bar{Q}\gamma^{\mu}\partial_{\mu}Q - m_Q\bar{Q}Q - \frac{1}{4}S^a_{\mu\nu}S^{a\mu\nu} + g_s J^a_{\mu}U^{a\mu} \, . \tag{1}$$

Here Q and U_{μ} represent the quark (color) triplet and gauge vector field. g_s is the strong coupling constant, $J^a_{\mu} = \bar{Q}T^a\gamma_{\mu}Q$ and T^a's belong to the SU(3) Gell-Mann matrices. The strength tensor is $S^a_{\mu\nu} = \partial_{\mu}U^a_{\nu} - \partial_{\nu}U^a_{\mu} + g_s f^{abc}U^b_{\mu}U^c_{\nu}$ with f^{abc} is the structure constant of SU(3) group respectively. It should be noted that the quarks and anti-quarks feel the electromagnetic force due to the U(1) field A_{μ}, but the size is suppressed by a factor of $e/g_s = \sqrt{\alpha/\alpha_s} \sim O(10^{-1})$.

Following the original model [30], the gluon fluid is put to have a particular form in term of relativistic velocity as,

$$U^a_{\mu} = (U^a_0, \mathbf{U}^a) \equiv u^a_{\mu}\,\phi \, , \tag{2}$$

with $u^a_{\mu} \equiv \gamma_{\mathbf{v}^a}(1, \mathbf{v}^a)$ and $\gamma_{\mathbf{v}^a} = \left(1 - |\mathbf{v}^a|^2\right)^{-1/2}$. $\phi = \phi(x)$ is a dimension one scalar field to keep correct dimension and should represent the field distribution. It is argued that taking this form leads to the equation of motion (EOM) for a single gluon field as follow [30],

$$\frac{\partial}{\partial t}\left(\gamma_{\mathbf{v}^a}\mathbf{v}^a\phi\right) + \nabla\left(\gamma_{\mathbf{v}^a}\phi\right) = -g_s \oint \mathrm{d}\mathbf{x}\, \left(\mathcal{J}^a_0 + F^a_0\right) \, , \tag{3}$$

where $\mathcal{J}^a_{\mu}$ is the covariant current of gluon field, and F^a_{μ} is an auxiliary function which can be found in the original paper [30]. It has been concluded that Eq. (3) should be a general relativistic fluid equation, since at the non-relativistic limit Eq. (3) coincides to the classical Euler equation.

More precisely, this fact provides a clue that a single gluonic field U^a_{μ} may behave as a fluid at certain scale, beside its conventional point particle properties with a polarization vector ϵ_{μ} in the form of $U^a_{\mu} = \epsilon^a_{\mu}\,\phi$. One can then consider that there is a kind of "phase transition",

$$\underbrace{\text{hadronic state}}_{\epsilon^a_{\mu}} \longleftrightarrow \underbrace{\text{QGP state}}_{u^a_{\mu}} \, . \tag{4}$$

As the gluon field behaves as a point particle, it is in a stable hadronic state and is characterized by its polarization vector. On the other hand in the pre-hadronic state (before hadronization) like hot QGP, the gluon field behaves as a highly energized flow particle and the properties are dominated by its relativistic velocity.

One should also recall that the wave function U^a_μ for a free massless particle satisfies $\partial^2 U^a_\mu = 0$ with a solution $U^a_\mu \sim \epsilon^a_\mu \exp(-ip_\nu x^\nu)$ where p_ν is the 4-momentum. Imposing the Lorentz guage condition $\partial^\mu U^a_\mu = 0$ demands $p^\mu \epsilon^a_\mu = 0$. Therefore the number of independent polarization vectors is reduced from four to three in a covariant fashion. However, one can still perform another gauge transformation to the massless U^a_μ which makes finally only two degrees of freedom remain. Therefore, one should keep in mind that in the present model the spatial velocity has only two degrees of freedom, that means one component must be described by another two vector components. Fortunately, in real applications in cosmology or compact star, this requirement is satisfied by the assumption that the system under consideration is isotropic.

On the other hand, let us comment on the gauge transformation of velocity u^a_μ. The gauge field U^a_μ is transformed as $U^a_\mu \to (U^a_\mu)' = U^a_\mu + 1/g_s\, \partial_\mu \theta^a + f^{abc}\theta^b U^c_\mu$ with the local gauge parameter $\theta^a = \theta^a(x)$. Following the Lorentz condition, θ^a also satisfies $\partial^2 \theta^a = 0$. If the solution is $\theta^a = i\kappa^a \phi \exp(-ip\cdot x)$ with κ^a constant and $\phi \sim \exp(-ip\cdot x)$, u^a_μ is transformed as $u^a_\mu \to (u^a_\mu)' = u^a_\mu + \kappa^a/g_s\, p_\mu + i f^{abc}\kappa^b u^c_\mu$. The invariant 4-velocity satisfies ${u'}^2 = 1 + \kappa(m_g/g_s + if_g) + O(\kappa^2) \sim 1 = u^2$ since the second and third terms are suppressed by $\kappa^a \sim \theta^a \ll g_s \sim O(1)$. f_g is the factor of summed colored gluon states. Therefore, one can take the form of Eq. (2) for a good approximation.

From now, throughout the paper let us focus only on the gluon sea of plasma. This means one should consider only the related gluonic terms in Eq. (1),

$$\mathcal{L}_g = -\frac{1}{4} S^a_{\mu\nu} S^{a\mu\nu} + g_s J^a_\mu U^{a\mu} \,. \tag{5}$$

3 Energy momentum tensor

Now we are ready to proceed with deriving the energy momentum tensor within the model. It should be pointed out that once the hot (high energy) QGP state is achieved, the system is assumed to be predominated by the classical motion rather than the quantum effects.

Therefore the total action of matter for non-gravitational fields in a general geometry of space-time $\mathcal{R}$ is $S_g = \int_{\mathcal{R}} \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}_g$, where g is the determinant of metric $g_{\mu\nu}$. The variation of S_g in the metric is given by $\delta S_g = -\frac{1}{2} \int_{\mathcal{R}} \mathrm{d}^4x \sqrt{-g}\, \mathcal{T}_{\mu\nu}\, \delta g^{\mu\nu}$. Since the energy momentum tensor density is,

$$\mathcal{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_g}{\delta g^{\mu\nu}} \,, \tag{6}$$

one obtains,

$$\mathcal{T}_{\mu\nu} = S^a_{\mu\rho} S^{a\rho}_{\ \ \nu} - g_{\mu\nu}\mathcal{L}_g + 2g_s J^a_\mu U^a_{\ \nu} \, . \tag{7}$$

It is clear that Eq. (7) is symmetric as expected to fulfill the Einstein gravitational EOM. The total energy momentum tensor $T_{\mu\nu}$ is given by integrating out Eq. (7) in term of total volume in the space-time under consideration. This means $T_{\mu\nu}$ is a result of bulk of gluons flow in the system.

Furthermore, in a general space-time coordinates, the components of energy momentum tensor determine the total energy density (T_{00}), the heat conduction ($T_{0i,i0}$), the isotropic pressure (T_{ii}) and the viscous stresses (T_{ij} with $i \neq j$) of the gluonic plasma. Of course, in this case the derivative ∂_μ inside the strength tensor $S_{\mu\nu}$ should be replaced by the covariant one, ∇_μ. Also, the energy momentum tensor satisfies the conservation condition, $\nabla_\mu \, \mathcal{T}^{\mu\nu} = 0$. Nevertheless, one can trivially conclude that the model induces non-zero viscosity since generally $T_{ij} \neq 0$ for $i \neq j$. From the experimental clues, however the size should be small such that it is always treated perturbatively in most hydrodynamics models [8–13].

Before going further to apply these results, one should determine the quark current J^a_μ in Eq. (7). This can be simply calculated by considering the EOM (Dirac equation) of a single colored quark (Q) or anti-quark ($\bar{Q}$) with 4-momentum p_μ. Since the solution of the EOM is $Q(p,x) = q(p)\exp(-ip\cdot x)$, one immediately gets $\bar{q}\gamma_\mu q = 4p_\mu$. Assuming that all colored quarks / anti-quarks have the same momenta and the velocity of gluons and quarks inside the gluon sea are homogeneity, approximately $J^a_\mu U^{a\,\mu} \propto 4p_\mu U^\mu = 4m_Q\phi$ since $u_\mu u^\mu = u^2 = 1$.

4 Equation of state

Now we are ready to consider the compact star interiors in the model, particularly before transforming itself into neutron star.

The whole compact star is commonly described as a static spherically symmetric space-time represented by Schwarzschild geometry. This means one deals with the relativistic gravitational equations for the interior of spherically symmetric plasma distribution in the core. In the region under consideration the presence of gluonic fields flow induces non-zero energy momentum tensor which is making up the star. This is the phase before the neutron star is getting mature. Starting from the stellar nebula made of hot plasma which is gradually getting colder as the hadronization occurs from the colder surface, while the inner core is still in pure hot plasma state.

As a consequence of the diagonal metric of Schwarzschild space-time, the model falls back to the perfect fluid without viscosity and heat conduction, *i.e.* $T_{0i} = T_{ij} = 0$ for $i \neq j$. Also, since the plasma distribution should be spherically isotropic, it is considerable to put $v_1 = v_2 = v_3 = v$ as constant for all colored gluons. This assumption is consistent with the degree of freedom counting discussed in the preceding section. Moreover, the vanishing off-diagonal components of the Ricci tensor, R_{i0}, actually forces the spatial

3-velocity of the fluid must vanish everywhere. Hence particular assumption for v_i is indeed not necessary. However, the gluon distribution still depends on the radius length, $\phi = \phi(r)$.

For the sake of simplicity one can put homogeneous gluon fields for all color states, *i.e.* $U^a_\mu = U_\mu$ for all $a = 1, \cdots, 8$. This yields,

$$\begin{aligned}\mathcal{T}_{\mu\nu} &= \left[8\, g_s\, f_Q\, m_Q\, \phi(r) + g_s^2\, f_g^2\, \phi(r)^4\right] u_\mu u_\nu \\ &\quad - \left[4\, g_s\, f_Q\, m_Q\, \phi(r) - \frac{1}{4} g_s^2\, f_g^2\, \phi(r)^4\right] g_{\mu\nu}\,, \end{aligned} \tag{8}$$

where f_g is the factor of summed colored gluon states from the structure constant f^{abc}, while f_Q is the factor of summed colored quark states from $J^a_\mu U^{a\mu}$. Remind that the energy momentum tensor for perfect fluid takes the form,

$$\mathcal{T}_{\mu\nu} = (\mathcal{E} + \mathcal{P})\, u_\mu u_\nu - \mathcal{P}\, g_{\mu\nu}\,. \tag{9}$$

Here $\mathcal{E}$ and $\mathcal{P}$ denote the density and isotropic pressure for single fluid field, each is related to the total density and pressure of the system through $\rho = \oint \mathrm{d}^4x\, \mathcal{E}$ and $P = \oint \mathrm{d}^4x\, \mathcal{P}$ respectively. Obviously, from Eqs. (8) and (9) one can obtain the density and pressure in the model as follows,

$$\begin{aligned} P(r) &= \int_{\beta_0}^{\beta_s} \mathrm{d}t \int \mathrm{d}V \left[4\, g_s\, f_Q\, m_Q\, \phi(r) - \frac{1}{4} g_s^2\, f_g^2\, \phi(r)^4\right] \\ &= \frac{4\, g_s\, f_Q\, m_Q}{T} \int \mathrm{d}V \left[1 - \frac{g_s\, f_g^2}{16\, f_Q\, m_Q} \phi(r)^3\right] \phi(r)\,, \end{aligned} \tag{10}$$

$$\begin{aligned} \rho(r) &= \int_{\beta_0}^{\beta_s} \mathrm{d}t \int \mathrm{d}V \left[4\, g_s\, f_Q\, m_Q\, \phi(r) + \frac{5}{4} g_s^2\, f_g^2\, \phi(r)^4\right] \\ &= \frac{4\, g_s\, f_Q\, m_Q}{T} \int \mathrm{d}V \left[1 + \frac{5\, g_s\, f_g^2}{16\, f_Q\, m_Q} \phi(r)^3\right] \phi(r)\,, \end{aligned} \tag{11}$$

at a finite temperature $\beta = 1/T$ in a 3-dimensional spatial volume V. Here, T_s and T_0 denote the core surface and inner core temperatures.

The proper spatial volume element for Schwarzschild geometry is $\mathrm{d}V = \sqrt{B(r)} r^2\, \sin\theta\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\varphi$ with radius r and two angles θ and φ in spherical coordinates. The solution for $B(r)$ is given by $B(r) = \left[1 - 2Gm(r)/r\right]^{-1}$ with $m(r) = 4\pi \int_0^r \mathrm{d}\bar{r} \rho(\bar{r})\, \bar{r}^2$ is the 'bare mass'. This generates the proper integrated mass $\tilde{m}(r)$ contained within a coordinate radius r inside the star. Anyway, the inner structure of star with Schwarzschild geometry is well known as Tolman-Oppenheimer-Volkoff (TOV) equation which relates density and pressure in a unique way [32,33].

Obviously, using Eqs. (11) and (10) one can construct certain EoS,

$$P(r) = w(r)\, \rho(r)\ , \tag{12}$$

where in contrast with the conventional cosmological models,

$$w(r) \equiv \frac{\int \mathrm{d}V \left[1 - \frac{g_s\, f_g^2}{16\, f_Q\, m_Q} \phi(r)^3\right] \phi(r)}{\int \mathrm{d}V \left[1 + \frac{5\, g_s\, f_g^2}{16\, f_Q\, m_Q} \phi(r)^3\right] \phi(r)} \; , \tag{13}$$

is not a constant. This is actually one of the important consequences in the present model. Once the field distribution $\phi(r)$ is determined one can obtain certain forms of density and pressure.

5 Pressure and density distribution

For the sake of convenience later on, let us redefine the radius r to be the dimensionless one, *i.e.* the ratio of core and compact star radius : $r \to r' \equiv r/r_{\mathrm{o}}$. Here r_{o} is the compact star radius. Then, the pressure and density in Eqs. (11) and (10) can be expressed as,

$$\begin{aligned} F_X(r') = {} & F_{X\,0} + 16\pi\, g_s\, f_Q\, m_Q\, {r_{\mathrm{o}}}^3 \frac{T_0 - T_s}{T_0 T_s} \\ & \times \int_0^{r'} \mathrm{d}r' {r'}^2 \sqrt{B(r')} \left[1 + k_X \frac{g_s f_g^2}{16\, f_Q\, m_Q} \phi(r')^3\right] \phi(r') \; , \end{aligned} \tag{14}$$

after integrating out the time component. F_X denotes the pressure P or density ρ for $X = P, \rho$ respectively, while $k_P = -1$ and $k_\rho = 5$. $F_{X\,0}$ represents the initial F_X. Taking $A_1(r') = \int_0^{r'} \mathrm{d}r' {r'}^2 \sqrt{B(r')}\phi(r')$, $A_2 = \int_0^{r'} \mathrm{d}r' {r'}^2 \sqrt{B(r')}\phi(r')^4$, $k_1 = 16\pi\, g_s\, f_Q\, m_Q\, {r_{\mathrm{o}}}^3$ and $k_2 = \pi\, g_s^2\, f_g\, {r_{\mathrm{o}}}^3$ yield,

$$F_X(r') = F_{X\,0} + \frac{T_0 - T_s}{T_0 T_s} \left[k_1\, A_1(r') + k_X k_2\, A_2(r')\right] \; . \tag{15}$$

Finally, by substituting the expressions for $m(r')$ and $B(r')$ into Eq. (15) one finds,

$$F_X(r') = F_{X\,0} + \frac{T_0 - T_s}{T_0 T_s} \int_0^{r'} \mathrm{d}r' \frac{\left[k_1 + k_X k_2 \phi(r)^3\right] \phi(r')}{\sqrt{1 - \frac{8\pi G}{r'} \int_0^{r'} \mathrm{d}\bar{r'} \rho(\bar{r'}) \bar{r'}^2}} \; . \tag{16}$$

Taking its derivative in term of r', Eq. (16) leads to an ODE as below,

$$\Lambda_1(r') F_X''(r') + T^2\, F_X'(r')^3 - \Lambda_2 T^2 {r'}^2\, F_X(r') F_X'(r')^3 - \Lambda_3(r') F_X'(r') = 0 \; , \tag{17}$$

where $T \equiv T_0 T_s/(T_0 - T_s)$ and,

$$\begin{aligned}
\Lambda_1(r') &= 2{r'}^5 \phi(r')^2 \left[k_1 + k_X k_2 \phi(r')^3\right]^2 \ , && (18)\\
\Lambda_2 &= 8\pi G \ , && (19)\\
\Lambda_3(r') &= \left[5{r'}^4 \phi(r')^2 + 2{r'}^5 \phi(r')\phi'(r')\right] \left[k_1 + k_X k_2 \phi(r')^3\right]^2 \\
&\quad + 6 k_X k_2 \, {r'}^5 \left[k_1 + k_X k_2 \phi(r')^3\right] \phi(r')^4 \phi'(r') \ . && (20)
\end{aligned}$$

Note that a prime in ϕ or F_X means a derivative in term of r'.

Nevertheless, one can also derive a "differential" EoS by taking the derivative of Eq. (16) in term of r' and eliminating $B(r')$, that is,

$$P'(r') = \frac{16\, f_Q\, m_Q - g_s f_g^2\, \phi(r')^3}{16\, f_Q\, m_Q + 5 g_s f_g^2\, \phi(r')^3}\, \rho'(r') \ . \quad (21)$$

Now let us solve the ODE in Eq. (17). The ODE can be solved analytically by taking an approximation of small compact star core, *i.e.* $r' \ll 1$. This approximation is quite natural since by definition the compact star cores should be small enough. Hence one can expand the ODE near the origin ($r' \to 0$) to obtain the solution order by order,

$$F_X(r') \sim \left. F_X(r') + F_X^{(I)}(r') r' + \frac{1}{2!} F_X^{(II)}(r') {r'}^2 + \frac{1}{3!} F_X^{(III)}(r') {r'}^3 + \cdots \right|_{r'=0} . \quad (22)$$

First of all, taking the first derivative on Eq. (17) in term of r' yields,

$$3T^2 {F_X^{(I)}}^2(0) F_X^{(II)}(0) = 0 \ , \quad (23)$$

since $\Lambda_1'(0) = \Lambda_3'(0) = 0$. Further derivatives up to 7th order give,

$$\begin{aligned}
F_X^{(III)}(0) &= \frac{1}{3 F_X^{(I)}(0)^2} \left[2\Lambda_2\, F_X(0) F_X^{(I)}(0)^3 - 6 F_X^{(I)}(0) F_X^{(II)}(0)^2\right] \ , && (24)\\
F_X^{(IV)}(0) &= \frac{1}{F_X^{(I)}(0)^2} \left[6\Lambda_2 F_X(0) F_X^{(I)}(0)^2 F_X^{(II)}(0) + 2\Lambda_2 F_X^{(I)}(0)^4 \right.\\
&\quad \left. - 2T^2 F_X^{(II)}(0)^3 - 6 F_X^{(I)}(0) F_X^{(II)}(0) F_X^{(III)}(0)\right] \ , && (25)\\
F_X^{(V)}(0) &= \frac{1}{3T^2 F_X^{(I)}(0)^2} \left[\Lambda_3^{(IV)}(0) F_X^{(I)}(0) + 36\Lambda_2 T^2 F_X(0) F_X^{(I)}(0)^2 F_X^{(III)}(0) \right.\\
&\quad + 72\Lambda_2 T^2 F_X(0) F_X^{(I)}(0) F_X^{(II)}(0)^2 + 82\Lambda_2 T^2 F_X^{(I)}(0)^3 F_X^{(II)}(0)\\
&\quad - 24 T^2 F_X^{(I)}(0) F_X^{(II)}(0) F_X^{(IV)}(0) - 18 T^2 F_X^{(I)}(0) F_X^{(III)}(0)^2\\
&\quad \left. - 36 T^2 F_X^{(II)}(0)^2 F_X^{(III)}(0)\right] \ , && (26)\\
F_X^{(VI)}(0) &= \frac{1}{3T^2 F_X^{(I)}(0)^2} \left[76\Lambda_2 T^2 F_X^{(I)}(0)^3 F_X^{(III)}(0) + 36\Lambda_2 T^2 F_X(0) F_X^{(I)}(0)^2 \right.\\
&\quad + 24\Lambda_2 T^2 F_X(0) F_X^{(I)}(0)^2 F_X^{(IV)}(0) + 522\Lambda_2 T^2 F_X^{(I)}(0)^2 F_X^{(II)}(0)^2
\end{aligned}$$

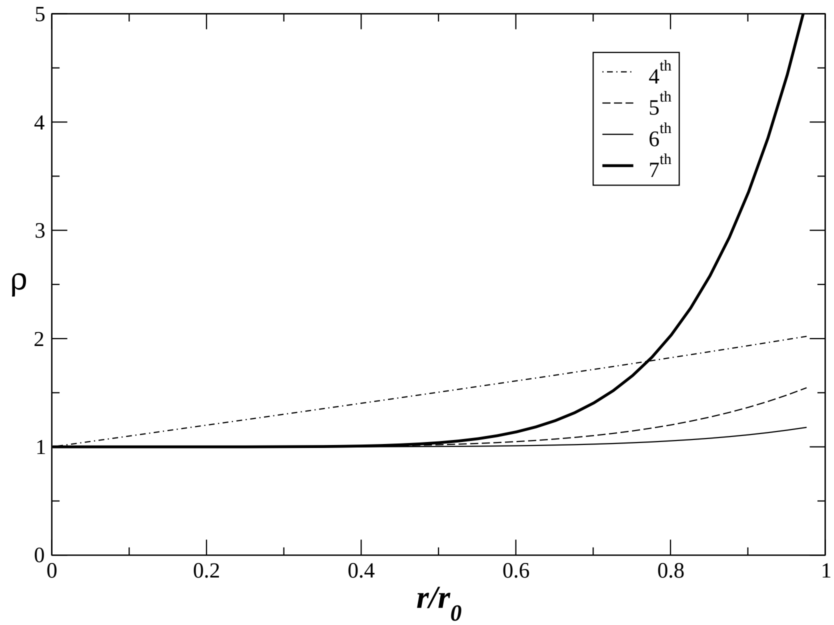

Fig. 1 The density distribution as a function of the normalized compact star core radius with $T_s = 175$ MeV and $T_0 = 1$ GeV up to 4^{th} (dotted-dashed line), 5^{th} (dashed line), 6^{th} (thin solid line) and 7^{th} (thick solid line) order expansions.

$$
\begin{aligned}
&+84\Lambda_2 T^2 F_X(0) F_X^{(II)}(0)^3 + 188\Lambda_2 T^2 F_X^{(I)}(0)^3 F_X^{(III)}(0) \\
&+342\Lambda_2 T^2 F_X(0) F_X^{(I)}(0) F_X^{(II)}(0) F_X^{(III)}(0) \\
&+36\Lambda_2 T^2 F_X(0) F_X^{(I)}(0)^2 F_X^{(IV)}(0) + \Lambda_3^{(V)}(0) F_X^{(I)}(0) \\
&+5\Lambda_3^{(IV)}(0) F_X^{(II)}(0) - \Lambda_1^{(V)} F_X^{(II)}(0) - 72T^2 F_X^{(II)}(0) F_X^{(III)}(0)^3 \\
&-60T^2 F_X^{(II)}(0)^2 F_X^{(IV)}(0) - 18T^2 F_X^{(II)}(0) F_X^{(III)}(0)^2 \\
&-60T^2 F_X^{(I)}(0) F_X^{(III)}(0) F_X^{(IV)}(0) \\
&-30T^2 F_X^{(I)}(0) F_X^{(II)}(0) F_X^{(V)}(0) \Big] \,, \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
F_X^{(VII)}(0) = &\frac{1}{3T^2 F_X^{(I)}(0)^2} \Big[378\Lambda_2 T^2 F_X^{(I)}(0)^3 F_X^{(IV)}(0) \\
&+66\Lambda_2 T^2 F_X(0) F_X^{(I)}(0)^2 F_X^{(V)}(0) + \Lambda_3^{(VI)}(0) F_X^{(I)}(0) \\
&+15\Lambda_3^{(IV)}(0) F_X^{(III)}(0) - 6\Lambda_1^{(V)}(0) F_X^{(III)}(0) \\
&-72T^2 F_X^{(III)}(0)^4 - 18T^2 F_X^{(III)}(0)^3 - 60T^2 F_X^{(I)}(0) F_X^{(IV)}(0)^2 \\
&-90T^2 F_X^{(I)}(0) F_X^{(III)}(0) F_X^{(V)}(0) \Big] \,. \qquad (28)
\end{aligned}
$$

From Eq. (23), non-trivial solution is obtained for $F_X^{(II)}(0) = 0$ which leads to $F_X^{(I)}(0) = \text{const}$. Without loss of generality, one can choose $F_X(0) =$

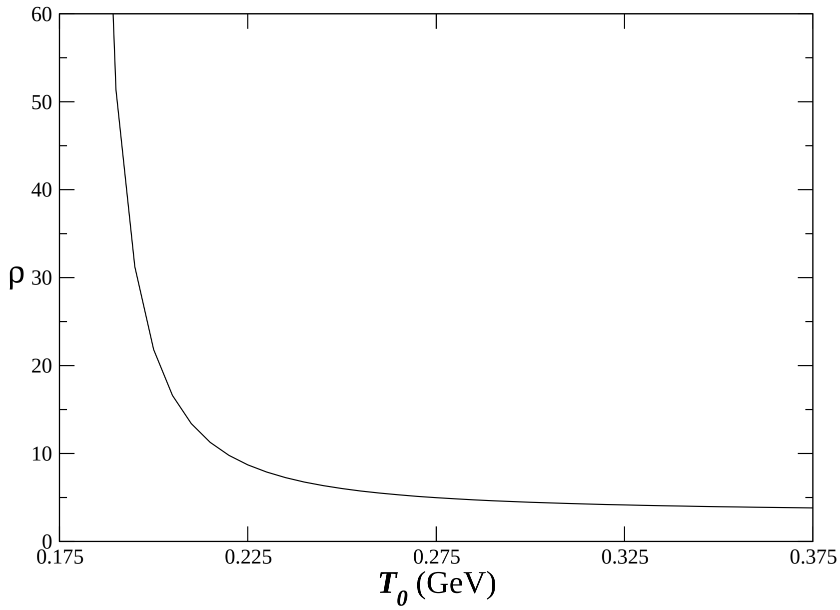

Fig. 2 The density distribution as a function of inner temperature with the normalized star radius $r' = 0.01$ and $T_s = 175$ MeV.

1. These results are substituted subsequently into Eqs. (24)∼(28) to get the complete solution. The solution is depicted in Figs. 1 and 2 for the density distribution, *i.e.* $F_X = \rho$.

It is important to derive the solution analytically to investigate the behavior of phase transition in term of higher order solutions. The results of total density as a function of r' are depicted in Fig. 1 for the solution up to 4^{th} (dotted-dashed line), 5^{th} (dashed line), 6^{th} (thin solid line) and 7^{th} (thick solid line) order expansions. It is clear that the phase transtion occurs at 7^{th} order accuracy. In particular the contour is mainly governed by the dissipative term, $\Lambda_3^{(VI)}(0)$. Those results are obtained by taking a particular form of distribution function, $\phi(r) \sim \exp(ar)$ with $a = 0.4$.

6 Summary

The QGP, in particular gluon matter, dominated compact start core interior has been investigated using the fluid QCD lagrangian. From the lagrangian one can derive the energy-momentum tensor and subsequently the density and pressure distributions from the first principle. In the model, both density and pressure are related each other and form an extended EoS of perfect fluid through the field distribution $\phi(r)$. The relation leads to the linear EoS known in the conventional cosmological model, but allowing the EoS parameter w to be a function of temperature T and core radius r. The analytical solutions for

the total pressure and density distribution equations are also given for small normalized compact star core radius, $r' \ll 1$.

The model describes the dense gluon plasma inside the compact star core as a bulk dynamics of gluonic matters before the cooling phase near the compact star core surface as depicted in Fig. 1. It should be noted that the figure provides a general contour for pressure or density distribution inside the core. This suggests that near its surface the hadronization is getting involved to form surfaces of small compact star cores,

On the other hand, the phase transition for certain size of compact star core in term of temperature changing is shown in Fig. 2. It can be observed that the phase transition occurs at the scale of hadronization around the core surface, while it keeps fluidity as the temperature is getting higher in the core interiors.

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